

# ***Measured crest height distribution compared to second order distribution***

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## ABSTRACT

When designing and verifying the structural integrity of an installation the loads that can affect the structure need to be determined. For the Norwegian continental shelf the governing regulations is given by the Petroleum Safety Authority. It states that the loads/actions that can affect the facilities shall be determined. Further, it states that accidental actions and metocean actions with an annual probability of  $10^{-4}$  or greater shall not result in loss of main safety functions.

In principle a full long term analysis is required in order to obtain consistent estimates for the metocean actions. This is straight forward for linear response problems, while it is a challenge for non-linear problems in particular if they additionally are of an on-off nature. The latter will typically be the case for loads due to wave in deck and breaking wave impacts.

In this paper, the second order distribution by Forristall will be discussed and compared to measurements(2004-2015) from a North Sea location at 190 m water depth. Long term analyses of crest height using short crested versus long crested waves will be discussed-

## I. INTRODUCTION

When designing a fixed drag dominated structure for wave loading a common approach is to use the so called “design wave approach”. This is normally specified by a long crested Stokes 5<sup>th</sup> order wave. The wave height with annual probability of  $10^{-2}$  is used for Ultimate Limit State (ULS). For Accidental Limit State (ALS) annual probability of exceedance of  $10^{-4}$  is used. The worst wave period within a 90 percent confidence interval is used, normally in the range +/- 2 s. If the structure responds dynamically to the wave loading, this has to be taken into account by adding the inertia load as an acceleration field. A structure is normally considered to respond dynamically to wave loading when the natural period of the structure is above 3 second.

In addition to the general wave loading, the airgap of installation has to be checked. If the extreme wave crest hits the deck. The solution will be to increase the air gap for new installations. For an existing installation the structural capacity has to be verified.

Thus the crest height with an appropriate annual probability has to be estimated. It should be noted that this crest will be higher than the crest given by the Stokes 5<sup>th</sup> design wave used in the design wave approach.

A common approach to estimate the extreme wave crest, used widely for the Norwegian continental shelf, is to perform a long term analysis utilizing the all sea state approach in combination with the short term second order distribution given by Forristall [1]

The crest distribution is a function of the wave steepness and the water depth. When the waves become very steep the crest height is limited by breaking. Extensive testing in model basin has been performed and there is an indication of increased crest heights beyond the second order crest even in directional spread sea, [2], [3], [4].

## II. SECOND ORDER WAVE CREST DISTRIBUTION

The sea surface elevation is now commonly described by a second order process. Based on second order time domain simulations, Forristall, [1] established a short term distribution of the crest heights. Key features of the numerical simulations performed by Forristall are:

- JONSWAP spectra.
- Peak enhancement factor 3.3(some 1.0 and 10).
- Water depths 10, 20, 40 m, and infinite.
- Peak periods 8, 10 and 12 s.
- Steepness  $S_p$  0.01 to 0.1, in steps of 0.01
- Simulation length 1024s, repeated 10 000 times.
- Long crested and short crested using Ewans spreading function for fetch limited sea,[5].

This distribution is commonly used to estimate the long term estimate of the extreme crests. The short term distribution is given by the following 2 parameters Weibull distribution:

$$F_C(x) = 1 - \exp \left[ - \left( \frac{x}{\alpha_C H_S} \right)^{\beta_C} \right] \quad (1)$$

Forristall fitted the Weibull location and scale parameter as a function of the water depth(Ursell number) and the wave steepness. He found that using the mean wave period( $T_1$ ), rather than the peak period produced a better fit for spectra with same peak period but different peak enhancement factors.

The steepness parameter is then given by:

$$S_1 = \frac{2\pi H_S}{g T_1^2} \quad (2)$$

Where  $T_1$  is the mean wave period estimated from the two first moment of the wave spectrum,  $m_0/m_1$ .

The effect of water depth is described by the Ursell number.

$$U_{rs} = \frac{H_S}{k_1^2 d^3} \quad (3)$$

Where  $k_1$  is the finite depth wavenumber for a frequency of  $1/T_1$  Note that the steepness is estimated using infinite water depth, but the wave number is estimated for finite water depth.

The estimated parameters are forced to match the Rayleigh distribution with  $\alpha = 1/\sqrt{8}$  and  $\beta = 2$  at zero steepness and Ursell number.

For long crested seas the Weibull parameters are given by:

$$\alpha_{C,2D} = 0.3536 + 0.2892S_1 + 0.1060U_{rs} \quad (4)$$

$$\beta_{C,2D} = 2 - 2.1597S_1 + 0.0968U_{rs}^2 \quad (5)$$

For short crested seas the Weibull parameters are given by:

$$\alpha_{C,3D} = 0.3536 + 0.2568S_1 + 0.0800U_{rs} \quad (6)$$

$$\beta_{C,3D} = 2 - 1.7912S_1 - 0.5302U_{rs} + 0.2824U_{rs}^2 \quad (7)$$

A stationary sea state of 3-hour duration is often considered. The 3-hour extreme crest distribution can then be estimated from:

$$F_{C3h}(x) = \left( 1 - \exp \left[ - \left( \frac{x}{\alpha_C H_S} \right)^{\beta_C} \right] \right)^{n_{3h}} \quad (8)$$

Where  $n_{3h}$  is the expected number of global crests in 3-hour. A global crest is largest crest between zero-up crossings.

The p-fractile for 3-hour can then be estimated from

$$x_p = \alpha_C H_S \left[ -\ln \left( 1 - F_{C3h}(x_p)^{\frac{1}{n_{3h}}} \right) \right]^{\frac{1}{\beta_C}} \quad (9)$$

The long term extreme crest should be estimated using a long term analysis. Alternatively, it can be estimated using the contour line method. The sea state at the peak of the contour and a fractile of 85%-90% for ULS and 90%-95% for ALS should then be used.

### A. Sensitivity to steepness and water depth

For jacket structures in the North Sea we are mainly interested in water depths of 80-190m, but to investigate the sensitivity to water depth and steepness a sensitivity analysis of the following cases is investigated. : Water depth from 80 to 1000 m.  $H_s = 10\text{m}$  and  $H_s = 18\text{ m}$ . Steepness varying from 0.01 to 0.1.

Steepness  $S_p$  is defined by

$$S_p = \frac{2\pi H_s}{g T_p^2} \quad (10)$$

Contours of constant probability of exceedance ( $q$ ) can be constructed for  $H_s$  and  $T_p$ . In deep water when the Ursell number approaches 0, the crest distributions will be the same in sea states that have the same steepness. Figure 1 show an example of contour lines and the steepness varying from 0.01 to 0.1. The green solid lines in Figure 1 indicate the area  $3.6 < T_p/\sqrt{H_s} < 5$ , where it is expected that the JONSWAP spectrum is a reasonable model according to [6]. This corresponds to steepness between 0.025 and 0.05. From the contour diagram we see that the highest steepness of 0.1 will only occur in low sea states. The peak of the contour is between 0.030-0.040.

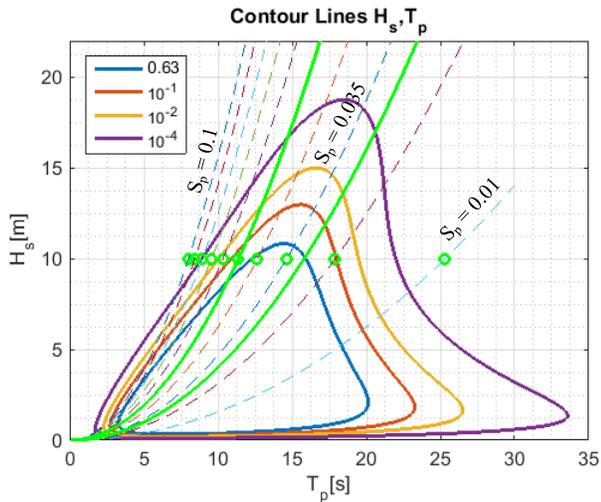


Figure 1 Example of contour lines of  $H_s$  and  $T_p$  with the steepness  $S_p$  shown as dashed lines.

In this sensitivity analysis it is assumed that the JONSWAP spectrum is a representative model. The mean wave period and the zero upcrossing period is estimated by the following relations taken from [6].

$$T_1 = (0.7303 + 0.04936\gamma - 0.006556\gamma^2 + 0.0003610\gamma^3)T_p \quad (11)$$

$$T_z = (0.6673 + 0.05037\gamma - 0.006230\gamma^2 + 0.0003341\gamma^3)T_p \quad (12)$$

The number of global wave crests is then estimated by

$$n_{3h} = \frac{10800s}{T_z} \quad (13)$$

The JONSWAP peak enhancement factor is estimated by the relation given by [7]

$$\gamma = \max[1, 42.2S_p^{6/7}] \quad (14)$$

The wave length is estimated by the following equation from [6].

$$\lambda = T\sqrt{gd} \sqrt{\frac{f(\omega)}{1 + \omega f(\omega)}} \quad (15)$$

where  $f(\omega) = 1 + \sum_{n=1}^4 \alpha_n \omega^n$ ,  $\omega = (4\pi^2 d)/(gT^2)$  and  $\alpha_1 = 0.666$ ,  $\alpha_2 = 0.445$ ,  $\alpha_3 = -0.105$ ,  $\alpha_4 = 0.272$ .

The target probability level for the characteristic value is varying for different sea states, since the number of crests is varying. The exceedance probability for the characteristic largest crest in 3-hour is  $1/n_{3h}$ , where  $n_{3h}$  is the expected number of crests during 3-hour. The characteristic largest crest is then given by:

$$c_n = \alpha_c H_s \left[ -\ln \left( 1 - \frac{1}{n_{3h}} \right) \right]^{1/\beta_c} \quad (16)$$

The characteristic largest crest height during 3-hour is estimated for sea states indicated by green circles in Figure 1 for 5 different water depths. The  $c_n$  is normalized by the significant wave height and shown in Figure 2 as a function of steepness.

From the figure we see that for water depth deeper than 100m that the Ursell number has little impact on the crest distribution compared to the steepness. It is also noted that the normalized characteristic largest crest increase with increasing steepness.

Sensitivity of Forristall distribution to water depth and steepness  
Long-crested,  $\gamma = \max(1, 42.2 S_p^{6/7})$ ,  $H_s = 10\text{m}$

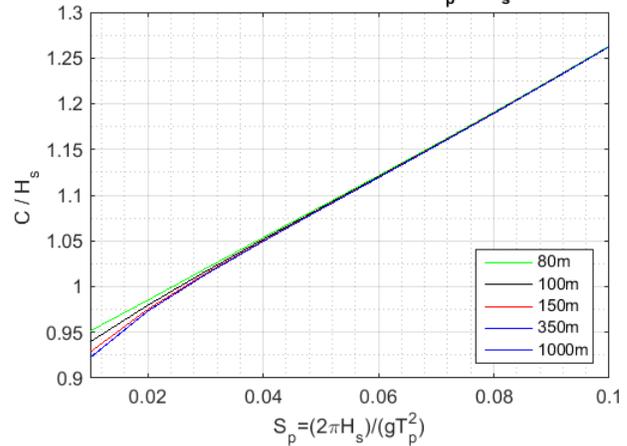


Figure 2  $C/H_s$  as a function of steepness for different water depths. Long crested waves.

The ratio between crest from 2D and 3D sea in a sea state of  $H_s=10m$  is shown in Figure 3. Shallow water and low steepness will produce waves that have higher crests in 3D sea than in 2D.

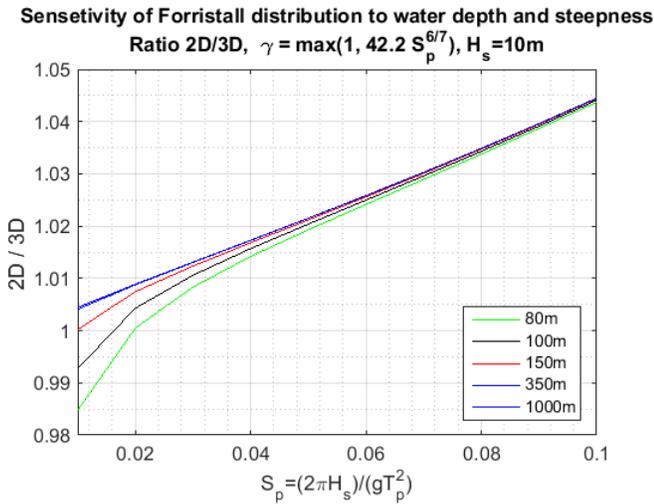


Figure 3 Ratio crests from long crested and short crested sea.

Further, three sea states at a water depth of 100m is studied in more detail. For very a very steep sea state,  $S_p=0.1$  at a sea state,  $H_s=10m$  the long rested waves will give slightly higher crests, see Figure 4. At the top of the contour the 2D and 3D are almost equal see Figure 5. For very long waves 3D becomes marginally higher than 2D, see Figure 6.

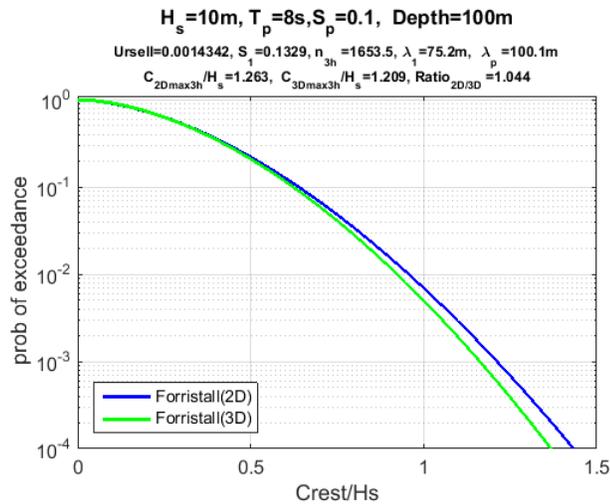


Figure 4 - 2D and 3D Forristall distribution in a very steep sea state,  $S_p=0.1$ .

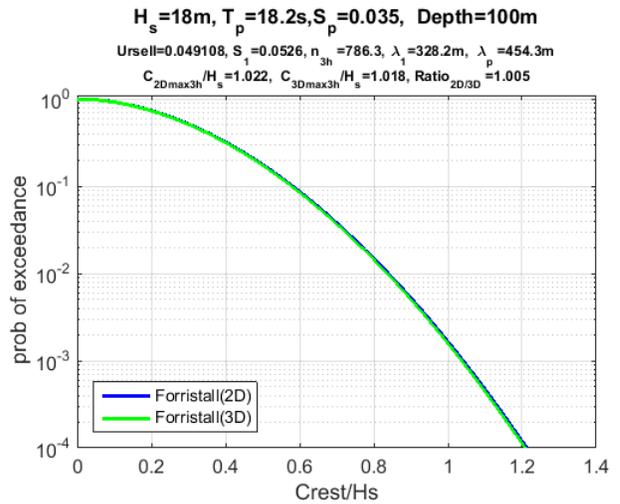


Figure 5 - 2D and 3D Forristall distribution at a steepness close to the peak of the contour,  $S_p=0.035$ .

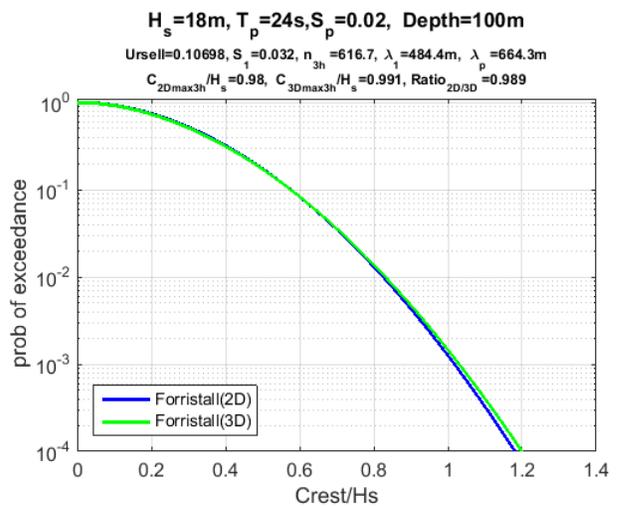


Figure 6- 2D and 3D Forristall distribution for long wave period,  $S_p=0.02$ .

III. FULL SCALE MEASUREMENTS

The measurement is performed by a wave radar type WaveRadar REX manufactured by SABB/Rosemount. The microwave beam has a 10 degree wide cone. The sampling frequency is 7.68Hz. A comprehensive review of wave radar measurements can be found in [8], [9].

Each time series has a length of 20 minutes. The mean is subtracted from each time series. Obviously erroneous time series has been removed.

The full scale measurement has been performed since 2004. There has been some down time, so the total available time series are 132752 x 20 min = 44250 hours. Of those 6348x20min = 2116 hours are in sea states above  $H_s$  6.5m, and 183x20min = 61 hours above  $H_s$  10.5m.

The highest measured 20 min. sea state is  $H_s=14.1m$ . The maximum crest in this sea state was 12.0m. The highest measured crest was 15.1m. This was in a sea state of  $H_s=12.1m$ .

A total of 2028 hours of measured surface elevation has been compared to the Forristall second order crest distribution. The comparison has in general been limited to sea state class with minimum 10 hours of observations. The measurements are binned into blocks of one meter and one second. Steepness from 0.018 to 0.055 are covered, as indicated by the shaded area in Table 1. The number of hours of measurements in each block is given in Table 2. The selected sea states are shown along with the contour lines in Figure 7.

Table 1 Steepness for the measured sea states are shown in shaded area.

		Tp[s]									
		9	10	11	12	13	14	15	16	17	18
Hs [m]	7	0.055	0.045	0.037	0.031	0.027	0.023	0.020	0.018	0.016	0.014
	8	0.063	0.051	0.042	0.036	0.030	0.026	0.023	0.020	0.018	0.016
	9	0.071	0.058	0.048	0.040	0.034	0.029	0.026	0.023	0.020	0.018
	10	0.079	0.064	0.053	0.045	0.038	0.033	0.028	0.025	0.022	0.020
	11	0.087	0.071	0.058	0.049	0.042	0.036	0.031	0.028	0.024	0.022

Table 2 Total hours of measurement of each sea state class.

		Tp[s]									
		9	10	11	12	13	14	15	16	17	18
Hs [m]	7	41.7	187.0	374.7	281.7	130.7	83.7	53.3	32.0	29.0	18.0
	8		32.3	149.7	175.3	57.0	33.0	17	13.3	16.7	
	9			27.7	61.7	58.3	25.3	13.3			
	10				23.7	37.0	27.3				
	11					13.7	14.0				

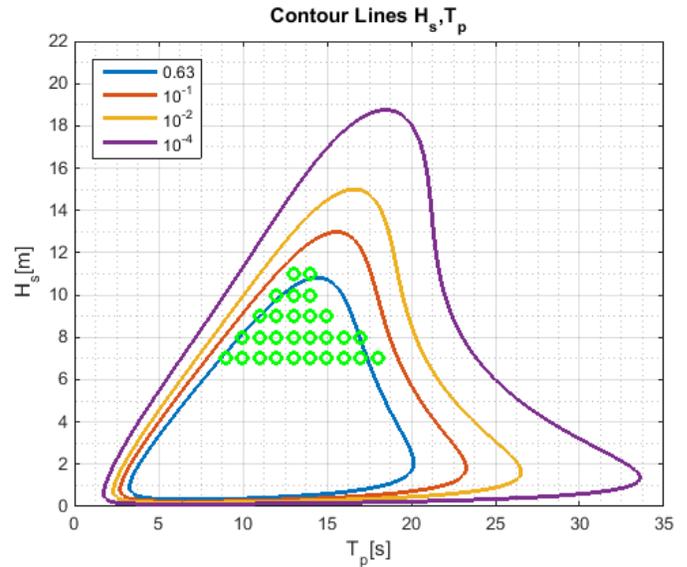


Figure 7 Sea states used for comparison with second order and full scale measurements.

In addition to comparing the visual plot of the measurement and second order distribution, the characteristic largest in a 3-hour sea states is compared.

The Gumbel extreme distribution is used to estimate the distribution of the largest:

$$F_{X|H_p T_p}(x) = \exp\left\{-\exp\left[-\left(\frac{x-\alpha}{\beta}\right)\right]\right\} \quad (17)$$

Where  $\beta$  is the scale parameter and  $\alpha$  the location parameter, [10].

The 3-hour mean and variance is given by:

$$E(X) = \mu_x = \alpha + 0.57722\beta \quad (18)$$

$$\sigma_x^2 = \frac{\pi^2}{6}\beta^2 = 1.64493\beta^2 \quad (19)$$

The location and scale parameters is estimated by the methods of moment (MOM), by solving equation (18) and (19).

There are several ways of estimating the characteristic largest in 3-hour from the measured 20minutes time series. One could for example randomly merge 9 time series and establish the Gumbel distribution of the largest. The characteristic largest can then be estimated by the most probably maximum (mpm) i.e the 37% fractile. This can be repeated several times and averaged.

We have chosen to establish the Gumbel distribution directly for the maximum in each 20 minutes series, the 3-hour distribution is then given by:

$$F_{X_{3h}|H_p T_p}(x) = \{F_{X_{20min}|H_p T_p}(x)\}^9 \quad (20)$$

The characteristic largest in 3-hour can then be estimated by the mpm of the Gumbel distribution, i.e the 37<sup>th</sup> percentile.

$$c_{n_{3h}G} = \alpha_{20min} - \ln\left(-\ln\left(0.37^{\frac{1}{9}}\right)\right)\beta_{20min} \quad (21)$$

This is compared to the characteristic largest estimated from the Forristall 3D distribution. To be consistent, we use the Gumbel extreme distribution instead of the true extreme distribution. For the 3-hour fractiles in the range of 37%-57% they give similar crests in this case. The deviation becomes larger further out in the tail. We estimate the Gumbel parameter from the asymptote of the largest observation from a Weibull distribution given by [10].

$$\alpha_{G|W} = \alpha_W (\ln(n))^{\frac{1}{\beta_W}} \quad (22)$$

$$\beta_{G|W} = \frac{\alpha_W}{\beta_W} (\ln(n))^{\frac{1-\beta_W}{\beta_W}} \quad (23)$$

Where  $\alpha_W$  and  $\beta_W$  are the Weibull scale and shape parameter respectively.  $n$  is the number of global crests in 3-hour. This can either be counted from the time series, or estimated by assuming JONSWAP spectra and equation (13). We have chosen the latter, in this comparison. The characteristic largest from second order distribution is then estimated by:

$$c_{n_{3h}G|W} = \alpha_{G|W} - \ln(-\ln(0.37))\beta_{G|W} \quad (24)$$

#### A. Results

In general the wave crest distribution is found to be very well described by the second order crest distribution by Forristall.

In Figure 8 the distribution from the sea state  $H_s$  7m and  $T_p$  11s. The variability in each 20-min sea state is indicated by the measured crest distribution for the different measurements, as indicated by the red circles. The coefficient of variation (COV) of the 20 min. extremes is estimated to  $COV=0.13$ , using  $\sigma/\mu$  from equation (18) and (19). The 3-hour COV is estimated to about 0.1. The merged crest distribution seems to be limited at the lowest probability; this could be due to breaking. The Gumbel distribution of the 20 minutes extremes is shown in Figure 9. This also indicates the limiting process out in the tail.

A sea state of less steepness is shown in Figure 10 and Figure 11. The crest distribution follows quite well the second order distribution to a probability level of about  $10^{-3}$ , then the measured crests slightly exceeds the Forristall distribution. The limiting process is less clear in this case.

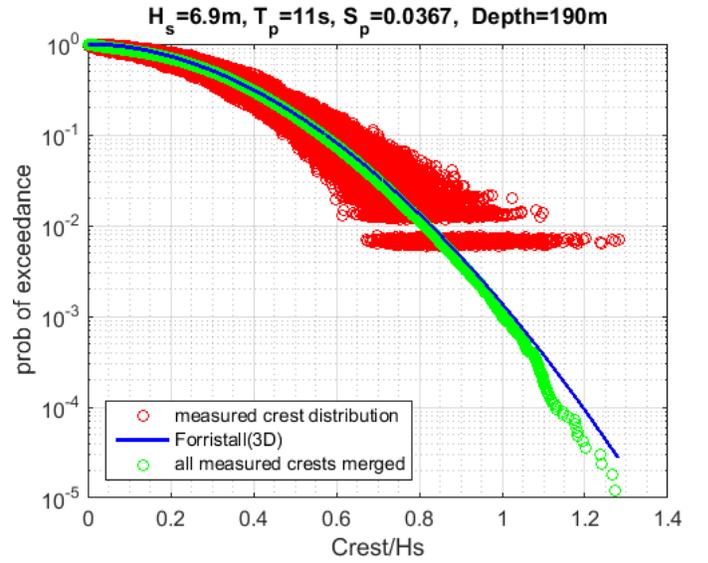


Figure 8 Steepness  $S_p=0.037$ . 375 hours of measurements.

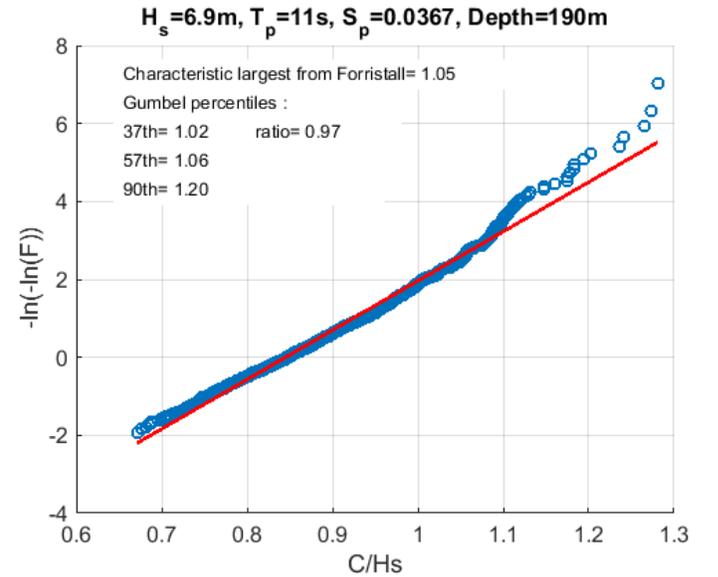


Figure 9 Gumbel distribution of 1124 , 20 minutes extremes.

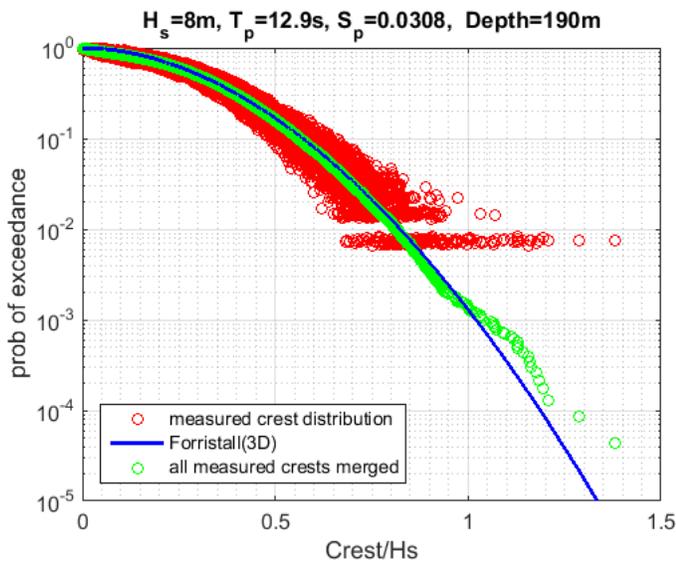


Figure 10 Steepness  $S_p = 0.031$ . 57 hours of measurements.

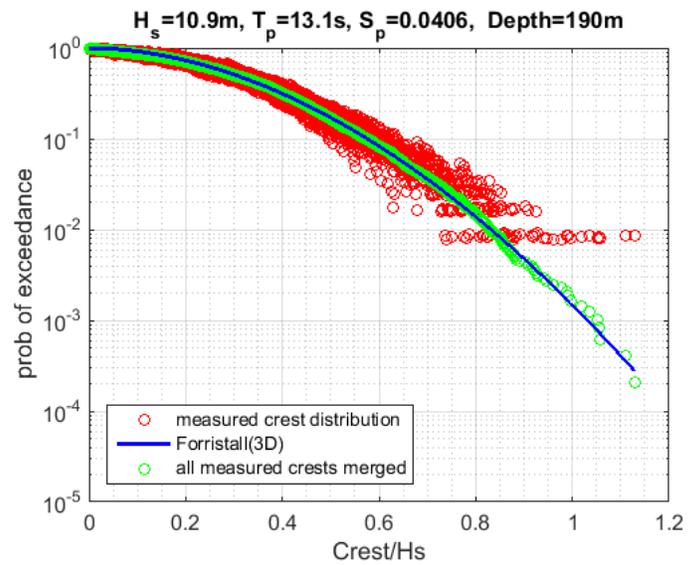


Figure 12 Steepness  $S_p = 0.011$ . 13.7 hours of measurements.

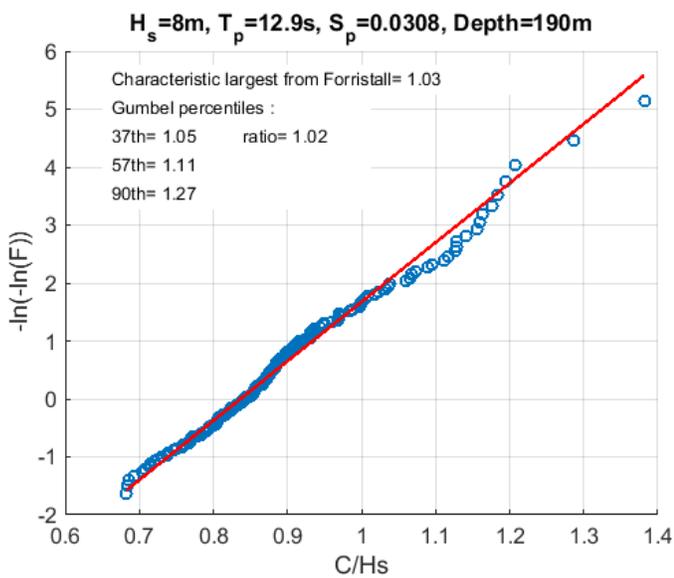


Figure 11 Gumbel distribution of 171 20 minutes extremes.

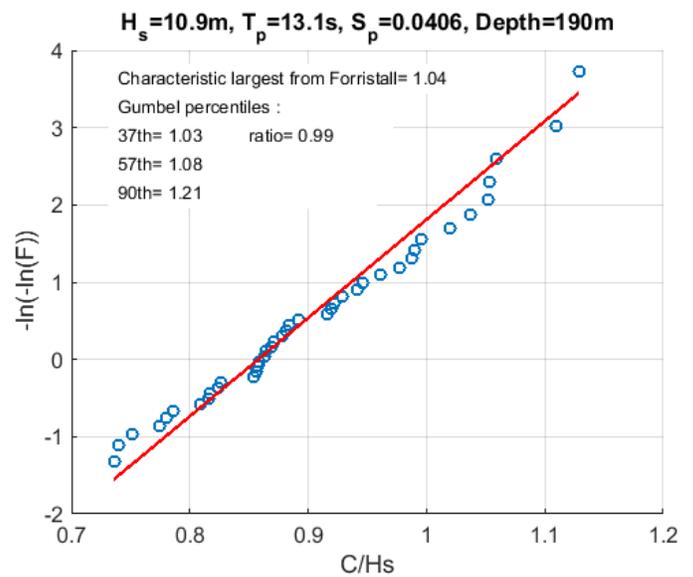


Figure 13 Gumbel distribution of 41 20 minutes extremes.

The distribution from the highest sea state class of  $H_s=11\text{m}$  is shown in Figure 12 and the Gumbel extreme distribution in Figure 13. The individual crest distribution follows very well the second order distribution. The largest crest in each 20min. follows the Gumbel extreme distribution.

The estimated characteristic largest crest in 3 hours divided by the estimated crest from second order distribution is shown in Table 3 and in Figure 14.

Table 3 Ratio between measured and second order characteristic largest in 3-hour

		Tp[s]									
		9	10	11	12	13	14	15	16	17	18
Hs [m]	7	0.97	0.97	0.97	0.99	0.99	1.00	0.99	1.00	1.02	1.03
	8		0.98	0.97	0.99	1.02	0.97	1.02	1.02	1.06	
	9			0.96	0.98	0.96	1.01	1.07			
	10				0.96	1.01	1.01				
	11					0.99	0.98				

A special storm the 26<sup>th</sup> of January 2012 lasting for more than 24 hours gave significantly higher crest distribution than second order, as shown by the red dot in Figure 14. The crest distribution is shown in Figure 15. For another data set, a similar significant deviation from the second order distribution is described in [11].

From Figure 14 there is a slope from the least steep towards the steep waves. This is somewhat linked to the number of waves. For the longest wave period there is a larger number up crossings than estimated using JONSWAP spectra, and for the shortest period, slightly smaller number than estimated. If the counted crests are used when estimating the characteristic largest crest from second order, instead of the estimated, the blue squares in Figure 14 will be slightly more level, but with the same tendency. The green squares indicate the change when using counted crests versus estimated at that steepness.

For the sea state class where the highest blue square is measured there is only 13.3 hour of measuring, and the next highest only 16.7 hours.

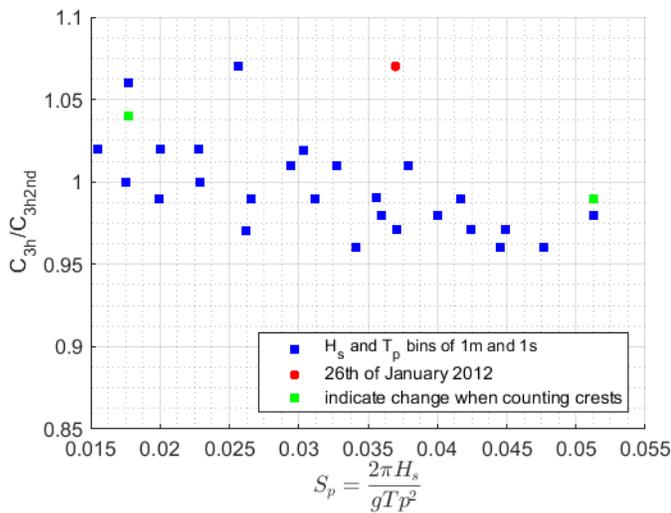


Figure 14 Ratio between measured and second order characteristic largest in 3-hour.

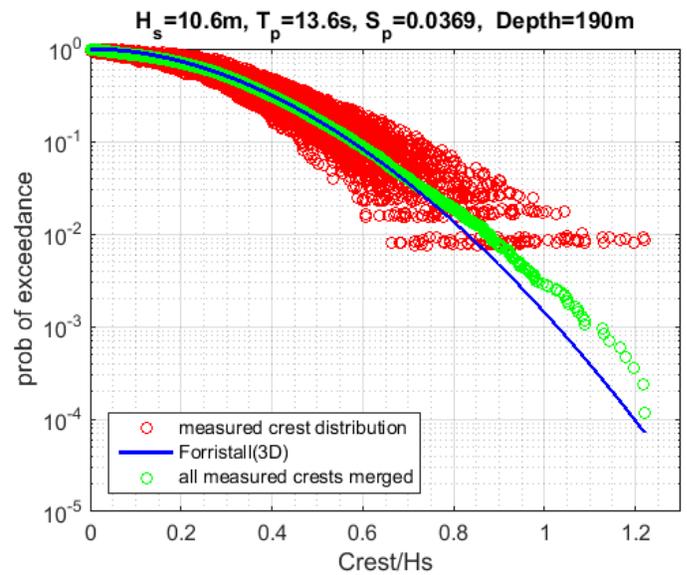


Figure 15 Crest Distribution 26 of January 2012. The characteristic largest is 7% higher than estimated by second order.

The time series during the 26<sup>th</sup> of January 2012 is shown in Figure 16. The storm is almost constant at a significant wave height of 11m for 24 hour. The Gumbel extreme value distribution of the 20 minutes extremes are shown in Figure 17. The characteristic largest crest during 3-hour is estimated to 7% higher than estimated from second order distribution.

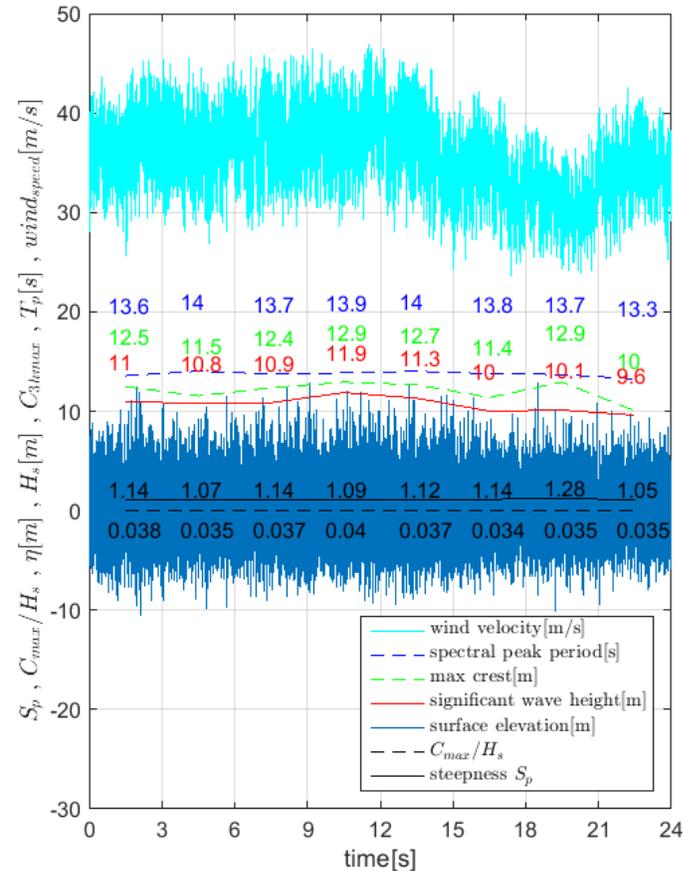


Figure 16 measured metocean condition during the 26<sup>th</sup> of January 2012.

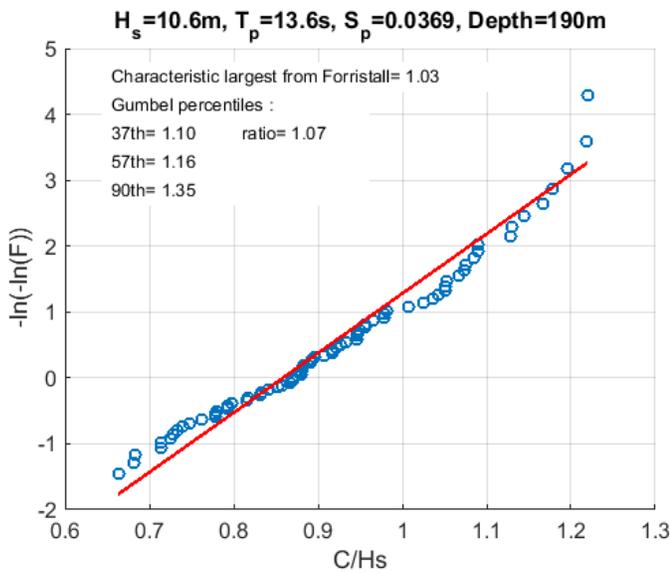


Figure 17 Gumbel plot of 20 min extremes. The percentiles are adjusted to 3hour extremes.

Each of the eight 3-hour distributions during the 26th of January 2012 are shown in Figure 18. It shows that for most of the 3-hour periods, the crests exceed the second order distribution.

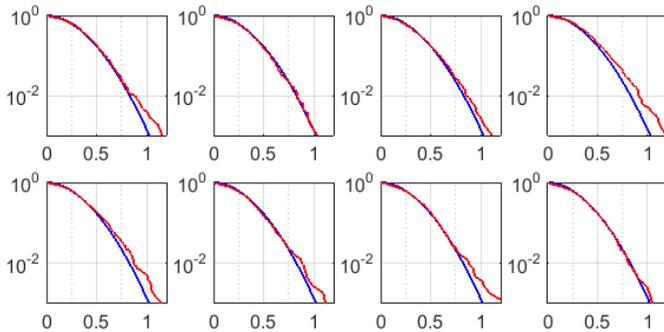


Figure 18 Each 3-hour crest distribution 26 of January 2012

The largest crest in the measuring period is 15m. This crest was measured in the evening of the 12<sup>th</sup> of January 2015. The crest distribution is shown in Figure 19. The time series around the extreme crest is shown in Figure 20, and the estimated wave spectra in Figure 21.

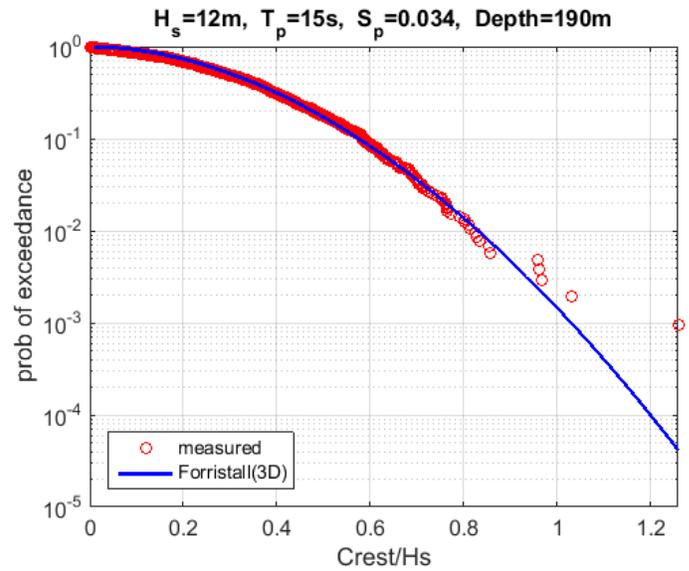


Figure 19 Crest distribution 12<sup>th</sup> January 2012 from 1800-2100

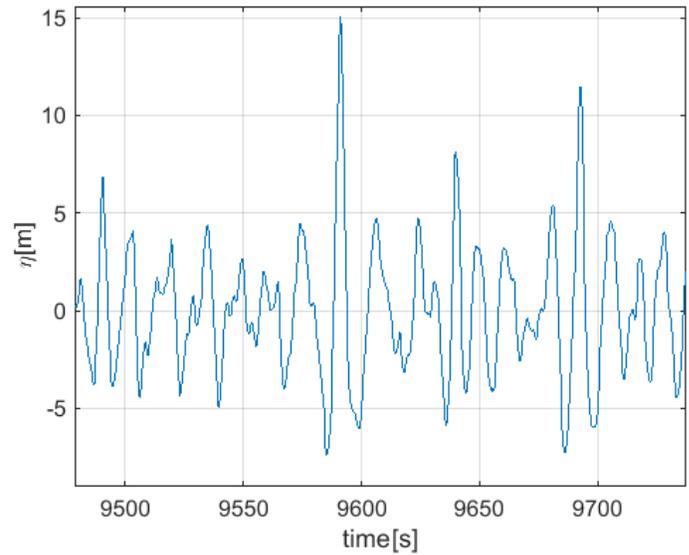
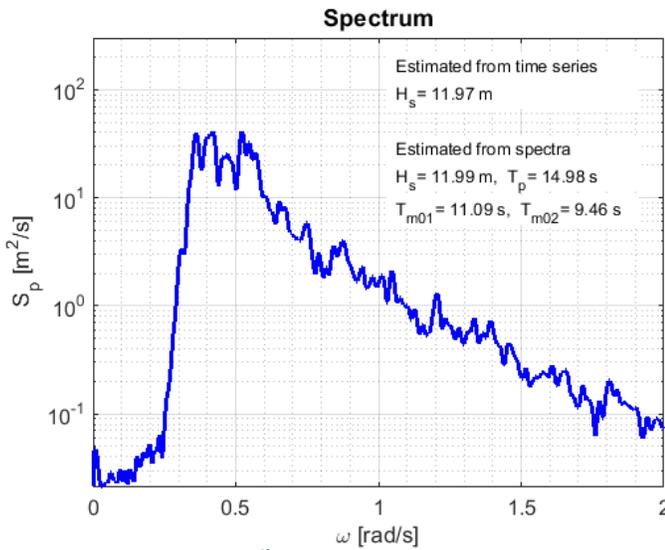


Figure 20 Extreme crest of 15 meter in sea state of Hs = 12m


 Figure 21 Wave spectra 12<sup>th</sup> january 2012 from 18:00-21:00.

#### IV. LONG TERM ANALYSIS USING ALL SEA APPROACH

We will now discuss long term analysis using all seas approach based on hindcast. When a long term analysis is performed using all seas approach, both the long – and short term inherent randomness is considered, Haver [12]. By considering the joint distribution  $f_{H_s T_p}(h_s, t_p)$ , of the significant wave height,  $H_s$ , and the spectral peak period  $T_p$ , the long term inherent randomness of the sea state severity is taken into account. A stationary sea state of 3hours duration is considered in the present work. The inherent randomness for the extreme response value in the 3-hour sea state is given by the short term conditional extreme value distribution  $F_{X_{3h} | H_s T_p}(x/h_s, t_p)$ .

The long term analysis (LTA) can be done by considering the 3-hour maximum as the target response quantity. The long term distribution of  $X_{3h}$  is given by:

$$F_{X_{3h}}(x) = \iint_{h,t} F_{X_{3h} | H_s T_p}(x/h_s, t_p) \cdot f_{H_s, T_p}(h, t) dt dh \quad (25)$$

The target annual exceedance probability  $q$  is then given by:

$$1 - F_{X_{3h}}(x_q) = q/m_{3h} \quad (26)$$

$m_{3h}$  is the annual number of events in the target population, if all 3-hours event in a year is included the target population is 2920. It is then assumed that all 3 hours extremes are statistically independent. This is not the case, and the estimated extreme value is expected to be slightly on the safe side. The short term distribution function of  $X_{3h}$  must be in agreement with the underlying physics of the response problem. The physics of the problem is related to the distribution of  $X_{3h}$ .

The long term description of the sea state can be estimated by fitting probability functions to the hindcast data.

The joint probability density function is given for  $H_s$  and  $T_p$  according to the following equation:

$$f_{H_s, T_p}(h_s, t_p) = f_{H_s}(h_s) \cdot f_{T_p | H_s}(t_p, h_s) \quad (27)$$

The long term distribution of the significant wave height is modeled by a 3 parameter Weibull distribution:

$$F_{H_s}(h_s) = 1 - \exp\left(-\left(\frac{h_s - \epsilon}{\theta}\right)^\gamma\right) \quad (28)$$

The significant wave for a given annual probability of exceedance can be estimated by

$$h_{s|q}(q) = \epsilon + \theta \left(-\ln\left(\frac{q}{2920}\right)\right)^{\frac{1}{\gamma}} \quad (29)$$

The probability density function for  $H_s$  is found by derivation of (28) :

$$f_{H_s}(h_s) = \frac{\gamma}{\theta} \left(\frac{h_s - \epsilon}{\theta}\right)^{\gamma-1} \cdot \exp\left(-\left(\frac{h_s - \epsilon}{\theta}\right)^\gamma\right) \quad (30)$$

Conditional distribution for  $T_p$  given  $H_s$ :

$$f_{T_p | H_s}(t_p | h_s) = \frac{1}{t_p \cdot \sigma \sqrt{2\pi}} \cdot \exp\left(-\frac{(\ln(t_p) - \mu)^2}{2 \cdot \sigma^2}\right) \quad (31)$$

where  $\mu = E[\ln T_p | H_s]$  and  $\sigma^2 = Var[\ln T_p | H_s]$  given by:

$$\mu = a_1 + a_2 h_s^{a_3} \quad (32)$$

$$\sigma^2 = b_1 + b_2 \exp(-b_3 h_s) \quad (33)$$

Table 4: Parameters in the annual omni-directional joint distribution for  $H_s$  and  $T_p$ .

$\gamma(shape)$		$\theta(scale)$		$\epsilon(location)$	
1.345		2.20		0.53	
$a_1$	$a_2$	$a_3$	$b_1$	$b_2$	$b_3$
1.653	0.397	0.395	0.005	0.096	0.283

When estimating long term crest height using all sea state approach and Forristall crest distribution, the long term distribution of the 3-hour maximum crest height is given by:

$$F_{C_{3h}}(x) = \iint_{h,t} \{F_{C | H_s T_p}(x)\}^{\frac{10800}{T_z}} \cdot f_{H_s, T_p}(h, t) dt dh \quad (34)$$

$T_z$  is the zero up crossings period. By assuming JONSWAP spectrum  $T_z$  is estimated by equation (12)

From the long term analysis we can see in Table 5 (column 2 and 3) that there is relatively little difference between the crest heights using 2D or 3D distribution. In column 6 the  $q$ -annual crest estimated by 3D distribution is normalized by the  $q$ -annual significant wave height. In column 7 the fractiles for the  $q$ -annual 3D crest is given.

Table 5 Long term results

Annual exceedance probability	q- 2D crest [m]	q- 3D crest [m]	H <sub>s</sub> [m]	T <sub>p</sub> [s]	$\frac{c_{q,3D}}{H_{s q}}$	$F_{C3h,3D}(C_q)$
0.63	12.0	11.8	10.8	14.4	1.09	72%
0.1	14.6	14.3	13.0	15.6	1.10	76%
0.01	17.1	16.8	15.0	16.6	1.12	82%
0.001	19.6	19.3	16.9	17.6	1.14	87%
0.0005	20.4	20.0	17.5	17.9	1.14	87%
0.0001	22.2	21.8	18.8	18.5	1.16	90%

## V. SUMMARY AND FURTHER WORK

In general the wave crest distribution is found to be very well described by the second order crest distribution by Forristall. From the estimated characteristic largest in 3-hour there is an indication of a slope from long waves towards steep waves. The largest deviation from the second order distribution is found in two sea state classes with relatively few hours of measuring.

However, during a storm that lasted for more than 24 hours, significantly higher crests than by second order were measured. From a different data set, a similar significant deviation from the second order theory is described in [11].

Although there are events exceeding second order distribution, it seems reasonable to assume second order wave distributions when performing a long term analysis.

An additional requirement could be to check that the probability of exceedance of the q-annual crest height should be less than a given percentage during a 3-hour storm.

From the long term analysis the estimated q-annual crest height of  $10^{-4}$  correspond to the 90% fractile in the sea state at the peak of the  $10^{-4}$  contour. If this is sufficient, need to be further studied. Further the effect of taking a larger area into account rather using a point estimate as addressed in [13] needs to be further discussed.

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